Statistics

Lecture 16

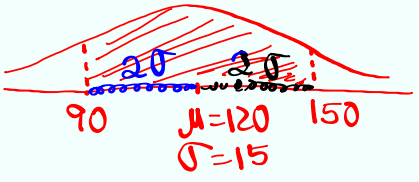
Feb 19-8:47 AM

Bonus Quiz

Given $N(120, 15)$

1) find $P(90 < x < 150)$ = normalcdf(90,150,120,15)

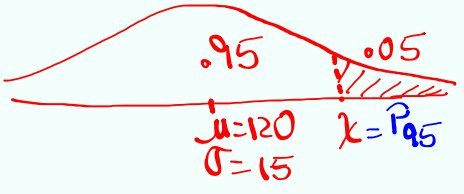
$= .954$ ✓ $\approx 95\%$ ✓



2) find $x = P_{95}$, Round to whole #.

x = invNorm(.95, 120, 15)

$= 144.673 \approx \boxed{145}$

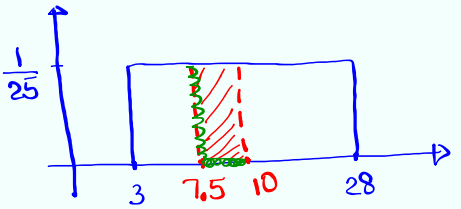


Apr 1-8:41 PM

Consider a uniform Prob. dist. for all values from 3 to 28.

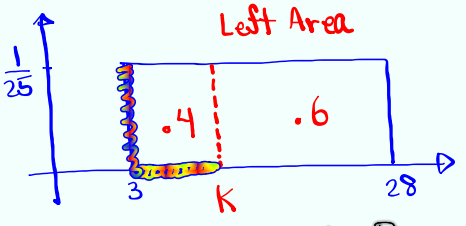
1) $P(x=5) = 0$

2) $P(7.5 < x < 10)$



$= (10 - 7.5) \cdot \frac{1}{25} = \frac{2.5}{25} = \frac{25}{10 \cdot 25} = \boxed{\frac{1}{10}}$

3) Find K such that $P(x \leq K) = .4$ ← Left Area



$(K-3) \cdot \frac{1}{25} = .4$

$K - 3 = 25(.4)$

$K - 3 = 10$

$\boxed{K = 13}$

$K = P_{40}$

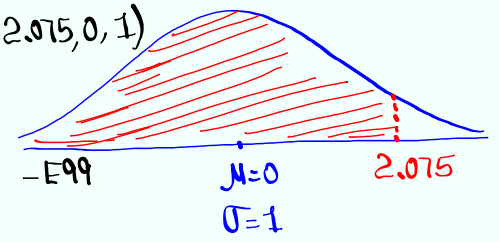
Apr 13-6:56 PM

Find $P(Z < 2.075)$

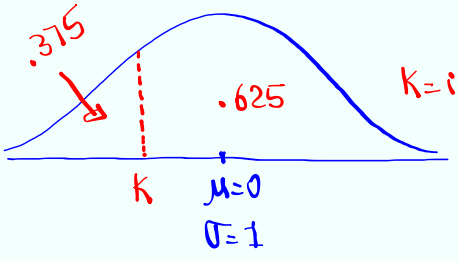
$N(0, 1)$

= normalcdf(−E99, 2.075, 0, 1)

= .981



Find K such that $P(Z > K) = .625$ ← Right Area



K = invNorm(.375, 0, 1)

$\approx \boxed{-.319}$

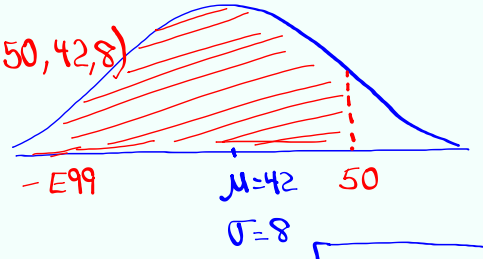
Apr 13-7:03 PM

Given $N(42, 8)$

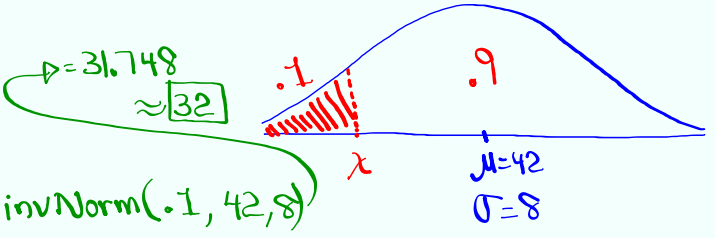
Find $P(x < 50)$

= normalcdf(−E99, 50, 42, 8)

≈ .841



Find x value that separates the bottom 10% from the rest.



x = 31.748 ≈ 32

x = invNorm(.1, 42, 8)

Apr 13-7:10 PM

Consider a normal Prob. dist. with the mean of 86 and Standard dev. of 8.

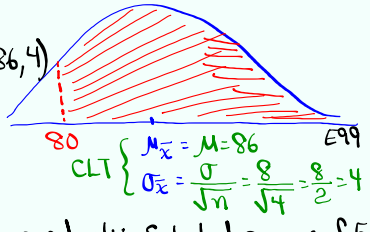
$N(86, 8)$

If we take Samples of Size 4 ($n=4$), find the prob. that their mean ($\bar{x}$) is above 80.

$P(\bar{x} > 80)$

= normalcdf(80, E99, 86, 4)

= .933

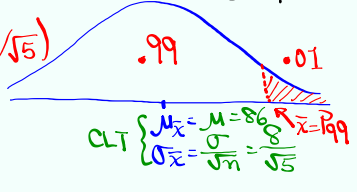


CLT $\mu_{\bar{x}} = \mu = 86$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{4}} = \frac{8}{2} = 4$

Find $\bar{x} = P_{99}$ for randomly Selected groups of 5.

$\bar{x}$ = invNorm(.99, 86, 8/√5)

= 94.323 ≈ 94



CLT $\mu_{\bar{x}} = \mu = 86$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{8}{\sqrt{5}}$, $\bar{x} = P_{99}$

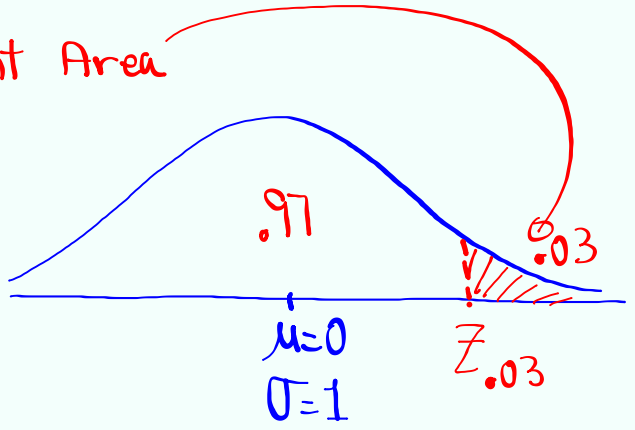
Apr 13-7:15 PM

find $z_{.03}$

Right Area

invNorm(.97, 0, 1)

= 1.881



Apr 13-7:26 PM

Estimating Parameter

Population → Parameter

Sample → Statistic

we use statistic with some level of confidence to estimate parameter.

estimation of parameter will be range of values.

Confidence Interval

Every conf. interval comes with confidence level $(1-\alpha)\cdot 100\%$

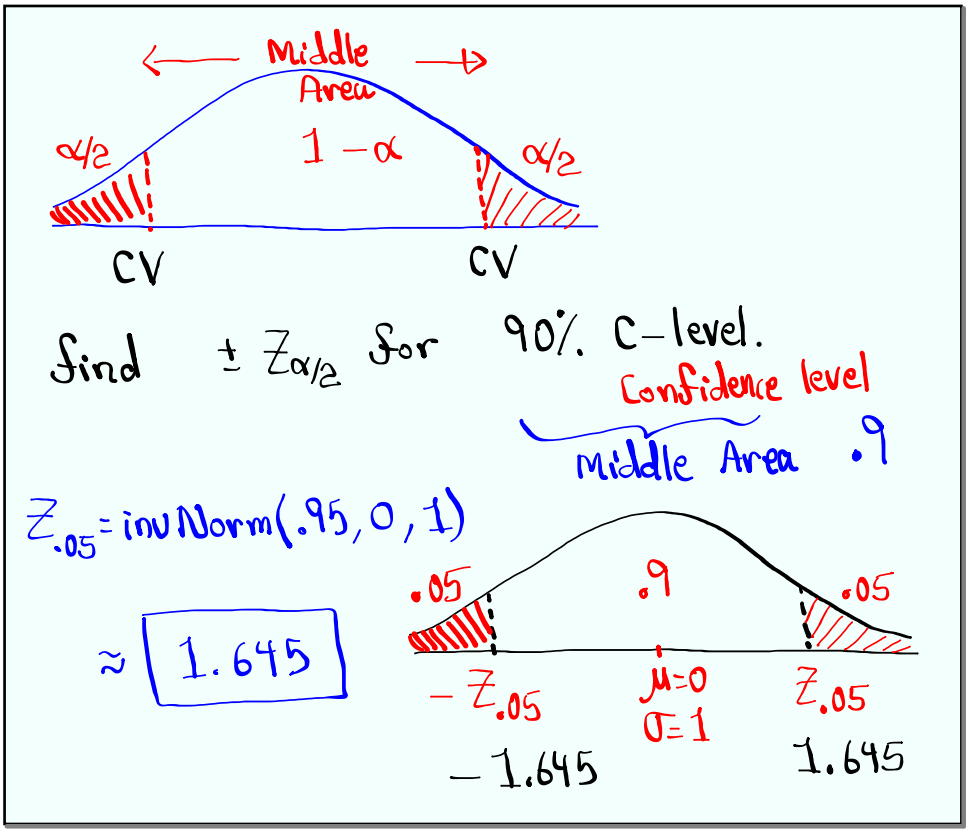
Alpha is significance level

$0 < \alpha < 1$

$\alpha/2$ is the area of each tail

$1-\alpha$ is the middle area

Apr 13-7:29 PM



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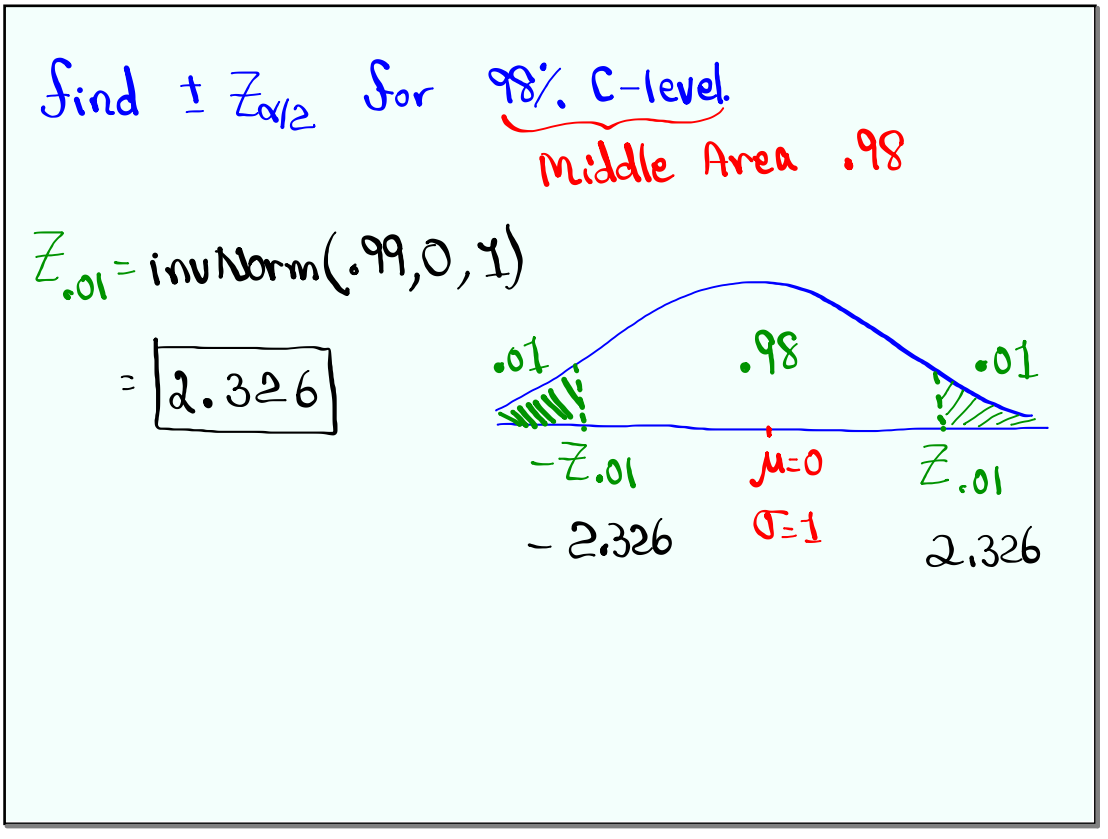


Apr 13-7:40 PM

Estimating Population Proportion

$$\hat{P} - E < p < \hat{P} + E$$

Margin of error

$$E = Z_{\alpha/2} \cdot \sqrt{\frac{\hat{P}\hat{q}}{n}}$$

Critical value for $(1-\alpha)\cdot 100\%$ C-level, Confidence level

$\hat{P}$ = P-hat (Point-estimate) Sample Proportion

$$\hat{P} = \frac{x}{n}$$

$$\hat{q} = 1 - \hat{P}$$

Apr 13-7:45 PM

I Surveyed 100 students and 80 of them had iPhone.

$n = 100$, $x = 80$

$\hat{P} = \frac{x}{n} = \frac{80}{100} = .8$ $\quad \hat{q} = 1 - \hat{P} = 1 - .8 = .2$

I want to find [90% Confidence interval] for the Prop. of all students that have iPhone. (Middle Area .9)

$$E = Z_{\alpha/2}\sqrt{\frac{\hat{P}\hat{q}}{n}}$$

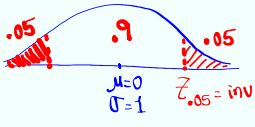
$$= 1.645\sqrt{\frac{(.8)(.2)}{100}} \approx .07$$

$\hat{P} - E < P < \hat{P} + E$

$.8 - .07 < P < .8 + .07$

$[.73 < P < .87]$

we are 90% Confident that between 73% & 87% of all students have iPhone.



Using TI:

STAT → TESTS 1 1-PropZInt

$.73 < P < .87$

$x = 80$
$n = 100$
C-level: .9
Calculate

$$E = \frac{.87 - .73}{2}$$

$$= .07$$

Apr 13-7:51 PM

I Surveyed 250 voters and 100 of them voted Yes on measure....

Find 99% Conf. interval for the proportion of all voters that voted Yes on measure....

$n = 250$

$x = 100$

C-level: .99

$E = \frac{.48 - .32}{2}$

$= .08$

$.32 < p < .48$

1-PropZInt

$x = 100$

$n = 250$

C-level: .99

we are 99% Confident that between 32% and 48% of all voters voted Yes on measure....

Apr 13-8:07 PM

In a Survey of 135 students, 62% of them had a laptop.

$n = 135$

$\hat{p} = .62$

$x = n\hat{p} = 135(.62) = 83.7 = 84$

if decimal → Round up

Find Conf. interval for the prop. of all Students that have laptop.

no Conf. level

use 95% C-level

$.54 < p < .70$

$E = \frac{.70 - .54}{2} = .08$

1-PropZInt

$x = 84$

$n = 135$

C-level: .95

we are 95% Confident that between 54% and 70%

$\hat{p} = \frac{.70 + .54}{2} = .62$ of all students have laptop.

Apr 13-8:14 PM

How to determine Sample Size:

$$E = z_{\alpha/2} \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

if we solve for n

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

if decimal → Always Round-up

If $\hat{p}$ and $\hat{q}$ unknown, use .5 for each,

$$n = .25\left(\frac{z_{\alpha/2}}{E}\right)^2$$

Apr 13-8:23 PM

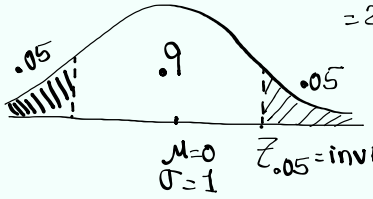
Find minimum Sample Size needed if we wish to construct 90% Conf. interval for pop. proportion with $\hat{p}=.6$ and $E=.05$.

$$n = \hat{p}\hat{q}\left(\frac{z_{\alpha/2}}{E}\right)^2$$

$$= (.6)(.4)\left(\frac{1.645}{.05}\right)^2$$

$$= 259.7784$$

$n = 260$



$z_{.05} = \text{invNorm}(.95, 0, 1)$

Redo with $E = .1$

$$n = (.6)(.4)\left(\frac{1.645}{.1}\right)^2 = 64.9476$$

$$\approx 65$$

Apr 13-8:27 PM

Redo using $E = .04$ and $\hat{p}$ & $\hat{q}$ unknown.

$$n = .25\left(\frac{1.645}{.04}\right)^2$$

$$= 422.816\ldots$$

$$\boxed{423}$$

Apr 13-8:35 PM

Estimating Population Mean

μ

$$\bar{x} - E < \mu < \bar{x} + E$$

$\bar{x}$: Sample Mean, Point-estimate

E: Margin of error

Case I: σ Known

$$E = z_{\alpha/2} \cdot \frac{\sigma}{\sqrt{n}}$$

STAT TESTS ZInterval

inpt: STATS

Follow the Screen

Apr 13-8:37 PM

Given $n=30$, $\bar{x}=84$, $\sigma=12$, C-level: .9

Find Conf. interval for pop. mean.

STAT TESTS ZInterval

$80 < \mu < 88$

inpt: Stats

$\sigma=12$

$\bar{x}=84$

$n=30$

C-level: .9

$E = \frac{88-80}{2}$

$= 4$

Apr 13-8:42 PM

I surveyed 40 students, their mean age was 26.5 yrs.

$n=40$, $\bar{x}=26.5$

It is known that standard dev. of ages of all students is 7.5 yrs.

$\sigma=7.5$

Find Conf. interval for mean age of all students.

$24.2 < \mu < 28.8$

no C-level use .95

σ known → ZInterval

inpt: Stats

$\sigma=7.5$

$\bar{x}=26.5$

$n=40$

C-level: .95

$E = \frac{28.8-24.2}{2} = 2.3$

$\bar{x} = \frac{28.8+24.2}{2} = 26.5$

Apr 13-8:47 PM